MATHEMATICAL COMPETENCIES AS DETERMINANTS OF SECONDARY SCHOOL ACADEMIC ACHIEVEMENT IN PHYSICS FOR SUSTAINABLE EDUCATION

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Abstract
The poor secondary school students' academic achievement in Physics as shown in the literature over the past decades has been attributed to poor acquisition of mathematical competencies that can enable them to do well in practical Physics. The need to examine mathematical competencies as determinants of Physics achievement becomes necessary for sustainable education. The study which adopted an ex-post facto research design was carried out in Nsukka education zone of Enugu state during the 2021/2022 academic session. The sample size for the study was 388 senior secondary school II Physics students in 6 randomly sampled single-sex secondary schools. Trained research assistants used interview results to fill the mathematical competency in practical Physics questionnaire (MCPPQ) for each student. Grades spreadsheet was used to collect students' senior secondary certificate examination result in Physics. Cronbach's alpha, which yielded a value of .85 was used to measure the internal consistency reliability of MCPPQ. Beta-weight was used to answer all the research questions while Chi-square statistic was used to test the hypotheses at .05 level of significance and 425 degrees of freedom. The result of the study as perceived by the students indicated that basic practical Physics mathematical operations, graphing skills, generation of readings from equations derived from experimental setup and deductions from the graph in decreasing magnitude positively influenced Physics achievement in external examination. Male and female students' models were similar in terms of path coefficients in the model. Hence, it was recommended that Physics teachers should improve upon students' mathematical competencies in practical Physics to boost their Physics achievement.

Keywords: Mathematics, Physics, gender, achievement and sustainable development.

Introduction

The global community has recognized the critical role of education in achieving sustainable development goals (SDGs). Education serves as a catalyst for economic and social transformations by equipping individuals with knowledge, skills and values necessary for personal growth and active participation in the development process. SDGs encompass a wide range of interconnected objectives, such as poverty eradication, environmental, social and educational sustainability. Achieving the educational sub-goal requires a holistic approach that addresses various dimensions of education. Educational

dimension can cut across subject areas and creating room for subject interaction. Such interaction nature has necessitated one subject area to be dependent on one another. In order words, one subject can be a determinant of success in another subject. For example, knowledge gained in Mathematics can be a determinant of meaningful learning in Physics.

In the context of secondary school education in Nigeria, one subject that has faced persistent poor achievement challenge is Physics. External examinations in Nigeria have consistently revealed poor achievement rates in Physics among secondary school students. This poor achievement raises concerns about the quality of Physics education and its implications for the attainment of sustainable development goals in Nigeria. Among the various factors affecting students' achievement in Physics, poor mathematical competencies emerge as a crucial factor. The West African examinations council, WAEC (2016, 2017, 2018 & 2019) in all its series examinations partly attributed poor achievement of students in Physics to poor application of mathematical concepts. The summary of the poor mathematical concepts of the Physics students in practical Physics as listed by WAEC included: (i) inability to read measurement from instruments to the required accuracy.(ii) inability to tabulate data correctly. (iii) inconsistency in decimal places. (iv) inability to plot graphs correctly (iv) poor determination of slopes and (v) over approximation of evaluated values. (vi) Inability to plot graph when values are negative. (vii) inability to express figures in standard form and (viii) inability to convert raw values using the given scale factor.

Mathematical competency is operationally defined in this study as the comprehension and application of mathematical concepts, principles and problem-solving strategies necessary for understanding and excelling in the use of the principles of Physics to solve educational problems in the classroom. In this study, it encompasses generation of readings from equations, application of basic mathematical operations indexed in practical Physics, graphing skills and making deductions from the graph. Theoretical generation of readings in practical Physics for secondary school students is the art of using equations derived from the experimental setup to obtain the values of variables that are set to be determined in an actual experimentation. The basic mathematical operations in practical Physics involve addition, subtraction, square, square-root, cube and cube-root. Others are inverse operations and logarithmic transformations and approximations: significant figures and decimal places. The graphing competency refers to manual plotting of the graph of the ordinate against abscissa, observing zero-zero origin with good scales on both axes. Deduction from graph is operationally defined as drawing the best line of fit, slope and intercept measurements. It also involves correct interpretation of slope, ordinate and abscissa intercept values for measurement of physical constants.

Students' gender has also been reported in literature to influence achievement in Physics. Gender refers to the social and cultural roles, expectations, and identities associated with being male or female within a given society. It encompasses the range of characteristics, behaviors, and expressions that are culturally attributed to masculinity and femininity (Butler, 1990). Understanding gender as a social construct highlights the

importance of examining its influence on various aspects of individuals' lives, including academic pursuits such as physics achievement. Gender equality in education is a fundamental aspect of sustainable development. It ensures that girls and women have equal access to education, compared to their opposite gender. Girls and women education do not only encourage their individual well-beings but also contribute to the overall development of the society at large. Educated women are more likely to participate in decision-making processes, promote health and well-being within their families and contribute to economic growth more than their uneducated counterparts. Moreover, despite significant advancements, gender disparities persist in many educational systems worldwide, hindering the attainment of sustainable development goals. The term "gender differentials" refers to the variations observed between genders in a particular area, such as physics achievement. It acknowledges the potential differences in achievement that exist between males and females within an academic circle. Perspectives on gender differentials in Physics achievement appear to lack a specific direction. Physics Education researchers like Anamezie and Nnadi (2019) indicated in their study that there was no gender differential in achievement of Physics students when the instructional stimulus was drill and practice mode of computer aided instruction, other studies other used varied instructional stimuli or opinions reported the existence of gender differentials (Ugwu, 2011; Wang & Degol , 2017; Stoet & Geary,2 018;Yakubu, 2021).

Achievement in Physics as a school subject hugely relies on mathematical principles and concepts for problem-solving and understanding complex phenomena. Mathematical knowledge provides the foundation upon which students build their understanding of Physics, enabling them to apply theoretical principles to practical situations effectively. Dirac (1939), a known physicist while explaining the relevance of mathematical knowledge to Physics understanding noted that the physicist had two methods of making scientific inquiry: (i) the method of experimentation and observation and (ii) the method of mathematical reasoning. According to Dirac, experimentation is just for collection of data and the latter enables one to infer results about experiments that have not been [fully] performed. The method of simulating results that have not been fully performed falls in line with the use of mathematical models or equations to predetermine the supposed experimental values, keeping some experimental attenuation constant. Ataide and Greca (2013) observed that Physics students should have a deep understanding of Mathematical skills and utilize them when it is necessary in Physics. Juan (2001) also supported the relevance of Mathematics to Physics understanding, by indicating that Mathematics had a fundamental role to pay in modern science, since every scientific theory was rooted on a mathematical model or law. Therefore, inadequate possession of sound mathematical skills can impede students' ability to achieve in Physics examinations.

Equations derived from mathematical principles allow students to describe and predict the behavior of physical systems. In mechanics, for example, secondary school students can be taught how to derive a mathematical equation that connects all the variables in an experimental setup using algebraic symbols. For illustration, in an

experiment to measure the mass of a uniform meterstick (M) in the laboratory using a knife-edge and set of masses, m. The centre of gravity, CG of the meterstick is measured in a real experiment when no mass hangs on it. By hanging m, at a constant length of 5cm from the zero-end of the meterstick, and balancing it horizontally, perpendicular distance a(cm) between m and knife-edge and b (cm) between the knife-edge and CG, an algebraic equation can be obtained for measurement of the values of 'a', 'b' and 'M' using the principle of moment of force. From algebra, ma=Mb. 50= a+b+5. M = (g). Hence, for different values of 'm' and 'a', M can be estimated and their average values obtained to represent M, experimentally. Theoretically, the values of 'a' and 'b' which the experiment seeks to measure can be obtained without measurement. Since 'a'= using algebra, the values of 'a ' for each value of 'm' can be calculated, given M in the last equation. The value of 'b' is calculable, given 'a'. What the researcher is arguing in the above example is that a little experimental measurement of the constant term like M in a mathematical equation or an experimental setup is needed to determine the values of varying parameters which experimentation seeks to achieve. This is achieved when the values of other connected variables in the experiment like 'm' are known, without fully performing the experiment. Once M is determined by direct measurement using lever or electronic balance, the values of m like 100g, 200g are substituted into the algebraic equation one at a time, the values of the variables: 'a' and 'b' which the experiment seeks to measure can be obtained through calculation. This procedure has the propensity to awaken the curiosity of the students in problem solving. In addition, it acts as a benchmark for assessing the precision of experimental results.

Algebra is the theoretical framework of this study. It plays a pivotal role in bridging the gap between mathematical knowledge and Physics achievement. Algebra is a branch of Mathematics that uses symbols to represent physical phenomena. Mordern algebra according to Kvasz (2006) was propounded by Cardano (1501-1576), who used a set of rules to solve equation of the third degree. It also encompasses the rules for manipulating symbols to solve equations and study physical phenomena. The use of algebraic equation to solve problems in practical Physics is applied in the mathematical aspects of practical Physics emphasized in this study.

To explore the application of mathematical knowledge in understanding Practical Physics, it is important to review existing literature on the topic. Several studies have demonstrated the positive relationship between mathematical knowledge and Physics achievement in secondary schools (Thompson & Steinberg, 2014; Smith, 2015; Ramirez &Ramos, 2016; Kapucu, et al., 2016; Johnson et al., 2017; Wu et al., 2017; Chen & Klahr, 2018; Brown & Lee, 2019; Hu et al., 2019; Li et al., 2021; Chen, et al., 2021). These studies highlight the significance of mathematical skills in conceptualizing and solving Physics problems, emphasizing the need for integrated approaches that promote the development of both mathematical and Physics competencies. However, the study conducted by Winter and Airey (2022) negated the earlier findings. The authors indicated that success in Mathematics can often mask a lack of conceptual understanding among

undergraduates. Since there is no uniformity in the results of earlier conducted studies, there is the need to study the relationship between mathematical knowledge and Physics achievement using structural equation model. The essence of structural equation model analysis, according to Schumacker and Lomax (2010) is to measure the extent to which the theoretical model is supported by sample data. The structural model allows a holistic analysis of the relationships between unobservable phenomena like graphing skill and observable phenomenon (Physics achievement) as they are enshrined in the structural model.

Statement of the Problem:

The Physics students' poor achievement in senior secondary school certificate examination (SSCE) as shown in the literature over the past decades has been adduced to poor mathematical skill. Mathematical skill including the use of algebraic expression is very common in Physics teaching and learning. The understanding of mathematical skill by the Physics students is an important tool for success in Physics, including practical Physics. The WAEC chief examiners have consistently reported that the Physics students had poor mathematical skills in practical Physics. It is against this backdrop that the present study is being carried out to measure the linear relationship between mathematical skills and achievement in SSCE Physics. Therefore, the educational problem of this study posed as a question is: how far does mathematical knowledge act as a cofactor of secondary school Physics students' achievement in senior secondary school certificate examination for sustainable development?

Purpose of the Study

The purpose of the study was to measure mathematical competencies as determinants of secondary school Physics achievement for sustainable development. The study specifically sought to develop a structural equation model for measurement of the:

1. Beta-weight of the path between 'generation of readings from equation derived from the experimental setup after direct measurements of some constants in the experiment' and achievement in Physics senior secondary school certificate examination in the combined model
2. Beta-weight of the path between 'application of basic mathematical operations in practical Physics' and achievement in Physics senior secondary school certificate examination in the combined model.
3. Beta-weight of the path between 'graphing skill' and achievement in Physics senior secondary school certificate examination in the combined model.
4. Beta-weight of the path between 'deductions from the graph' and achievement in Physics senior secondary school certificate examination in the combined model.
5. Existence or otherwise of a significant difference between beta-weights in the observed and expected structural equation model of male Physics students.

6. Existence or otherwise of a significant difference between beta-weights in the observed and expected structural equation model of female Physics students.
7. Existence or otherwise of a significant difference between male and female students' beta-weights in the combined structural equation model.

Research Questions

The following research questions guided the study:

1. What is the beta-weight of the path between 'generation of readings from equation derived from the experimental setup after direct measurements of some constants in the experiment' and achievement in Physics senior secondary school certificate examination in the combined model?
2. What is the beta-weight of the path between 'application of basic mathematical operations in practical Physics' and achievement in Physics senior secondary school certificate examination in the combined model?
3. What is the beta-weight of the path between 'graphing skill' and achievement in Physics senior secondary school certificate examination in the combined model?
4. What is the beta-weight of the path between 'deductions from the graph' and achievement in Physics senior secondary school certificate examination in the combined model?

Hypotheses

The following null hypotheses tested at .05 level of significance guided the study:

$H0_1$: There is no significant difference between beta-weights of the paths in the observed and expected structural equation model of male Physics students.

$H0_2$: There is no significant difference between beta-weights of the paths in the observed and expected structural equation model of female Physics students.

$H0_3$: There is no significant difference between male and female students' structural equation models' beta-weights in the combined model.

Method

Ex-post facto also known as causal comparative research design was employed in the study. The choice of the design was because there was no manipulation of the variables and data in the study. The study was carried out in Nsukka education zone of Enugu State, Nigeria during 2021/2022 academic session. The population for the study consists of 12,672 students in 31 public secondary schools in Nsukka education zone (Statistical unit, Post primary schools Management Board, Enugu state, 2022). The sample for the study was 388 consisting of 204 female and 184 male Physics students. Simple random sampling technique, precisely balloting with replacement was used to sample two schools each from the three Local Government Areas under the education zone: Igbo-Etiti, Nsukka and Uzo-Uwani. Balloting with replacement was also used to sample only one stream of senior

secondary school three Science class in the six schools. There was unequal number of the students in the sampled schools. Using purposive sampling technique, three least populated schools were included. The remaining number was sampled using simple balloting with replacement. Purposive sampling technique was adopted to make sure that the least populated school had a chance of being included in the study. Interview was used as a primary source of data collection. Ten research assistants who were regular graduate Physics teachers were trained on how to collect data that were related to mathematical competencies in practical Physics. They were recruited as research assistants in the study because they showed evidence of having become WAEC examiners in Physics for at least two years. During the interview sessions which lasted for one month, students were scored 1 mark for correct display or answer to a question and 0 mark for non-possession of the needed mathematical competency. Mathematical competency in practical Physics questionnaire (MCPPQ) was used to collect data from the students by the research assistants during the interview. Students' scores ranging from 1-25%, 26-50%, 51-75% and 76-100% attracted 1, 2, 3 and 4 points respectively. Grades sheet was used to collect the students' year 2022 West African senior secondary certificate examinations (WASSCE) results in Physics, when they were released. MCPPQ was developed by the researcher. It had thirty-items with four-point rating scale ranging from 4(*Very High Extent)* to 1(*Very Low Extent).* MCPPQ had two sections: A and B. Section A sought information on demographic factors of the respondents while section B had four clusters: Application of basic mathematical operations in practical Physics for sustainable development, Generation of readings from equation derived from the experimental setup after direct measurements of some constants in the experiment for sustainable development, Graphing skills for sustainable development and Deductions from the graph for sustainable development. The original MCPPQ which had 54 items was both face and construct validated prior to its usage. The face validation targeted clarity, appropriateness and structure (positively and negatively worded) of the wordings of MCPPQ, whereas exploratory factor analysis (EFA) was used to measure the alignment of the factors of MCPPQ unto latent constructs. The preliminary result of the EFA showed the presence of 4 constructs with valid 30 items. The internal consistency reliability of MCPPQ was measured using Cronbach's alpha. Its value was .85. The students' SSCE results in Physics were collected using researcher-developed grades spreadsheet. The SSCE Physics grades were rated by West African Examinations Council, WAEC on nine-point inverse-stanine grades: A_1, B_2, B_3, C_4, C_5, C_6, D_7, D_8 and F_9. The researcher reversed the scoring of the collected grades to ensure that (i) higher grades attracted higher scores and (ii).equality of WAEC grades and MCPPQ rating scale was achieved as follows: 4(A_1-$B_{3)}$, 3(C_4-C_6), 2(D_7-D_8) and 1(F_9). The structural equation model based on the four latent constructs of MCPPQ was developed with their respective indicators using analysis of moment structures computer programme version 20. The path between each of the four latent constructs of MCPPQ and SSCE Physics grade was sought in the model. The data submitted for analysis did not have any case of missing data. Maximum likelihood

estimation technique was adopted in the structural equation model parameters' estimations. The reason was because it had higher precision and accuracy relative to many model parameter estimators. Beta-weights were used to answer the research questions while the null hypotheses were tested at .05 level of significance using Chi-square statistic. For the research questions, the interpretation of regression coefficients is as follows: 0.1-0.3(*low positive relationship),* 0.40-0.6 (*moderate positive relationship*) and 0.7-0.9 (*high positive relationship*) and vice versa. The decision rule for the hypothesis is that if the estimated probability, p value attached to the Chi-square value with its degree of freedom is less than the .05 level of significance set for the study, the null hypothesis is rejected. Similarly, if the estimated probability, p value is more than the .05 level of significance set for the study, the null hypothesis of no significant difference is accepted.

Results

Research Question 1:

What is the beta-weight of the path between 'generation of readings from equation derived from the experimental setup after direct measurements of some constants in the experiment' and achievement in Physics senior secondary school certificate examination in the combined model?

Table 1 below shows that the beta-weight of the path between generation of readings from equations from practical Physics setup and SSCE Physics achievement in the direction shown by the arrow. The estimated beta-weight of the path is 0.12.

Table 1: The coefficient of linear regression between the path: Physics achievement and generation of readings from equations in practical Physics in the combined model.

Linear Regression path	Beta-weight
SSCE Physics Achievement and Generation of readings from equations in practical Physics	0.21

This is a low positive relationship between the variables being measured. Figure 1 shows the sample of the evaluated structural equation model for combined male and female subgroups.

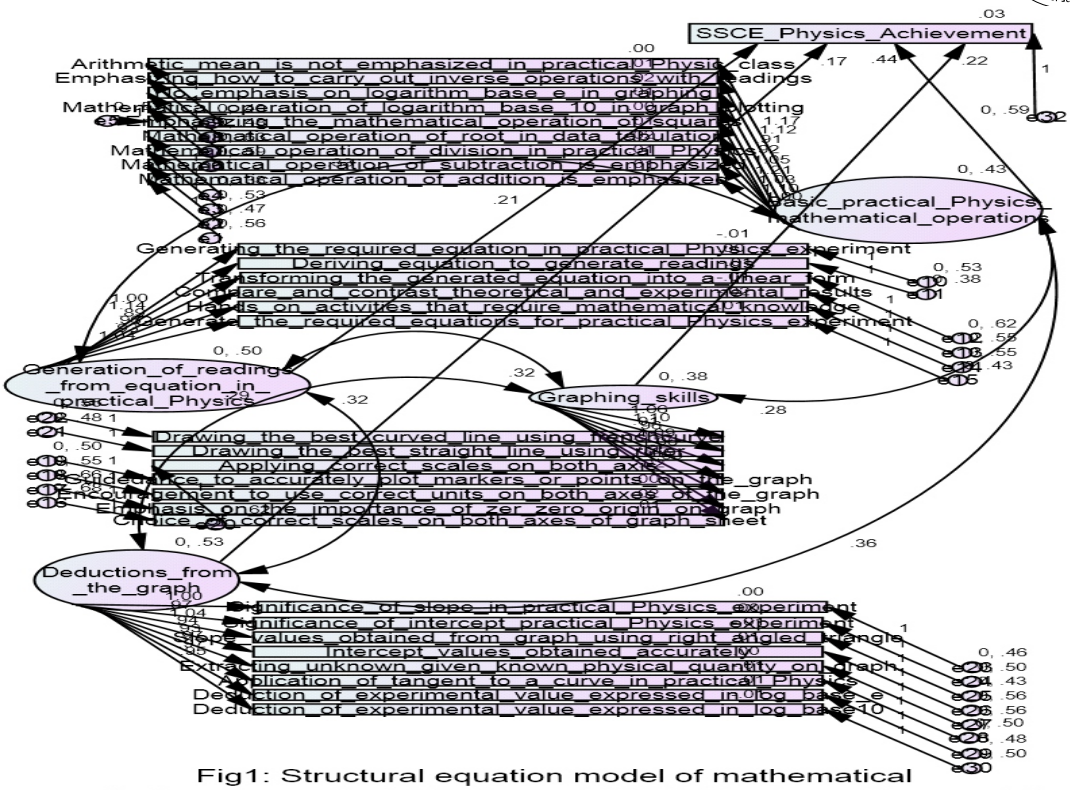

Fig1: Structural equation model of mathematical applications in practical Physics and SSCE Physics achievement for combined subgroups

Research Question 2:

What is the beta-weight of the path between 'basic practical Physics mathematical operations' and achievement in Physics senior secondary school certificate examination in the combined model?

Table 2 below shows the beta-weight of the path between basic practical Physics mathematical operations and SSCE Physics achievement in the direction shown by the arrow. The estimated beta-weight of the path is 0.44.

Table 2: The coefficient of linear regression between the path: Physics achievement and basic practical Physics mathematical operations in the combined model.

Linear Regression path	Beta-weight
SSCE Physics Achievement and Basic practical Physics mathematical operations	0.44

The value of the path coefficient shows that a moderate positive relationship between SSCE Physics Achievement and basic practical Physics mathematical operations.

Research Question 3:

What is the beta-weight of the path between 'graphing skill' and achievement in Physics senior secondary school certificate examination in the combined model?

In Table 3 below, the beta-weight of the path between graphing skill in practical Physics and SSCE Physics achievement in the direction shown by the arrow has a that is value equal to 0.22.

Table 3: The coefficient of linear regression between the path: Physics achievement and graphing skill in practical Physics in the combined model.

Linear Regression path	Beta-weight
SSCE Physics Achievement and Graphing skill in practical Physics	0.22

The value of the path coefficient shows that a low positive relationship between SSCE Physics Achievement and graphing skill in secondary school practical Physics exists.

Research Question 4:

What is the beta-weight of the path between 'deductions from the graph' and achievement in Physics senior secondary school certificate examination in the combined model?

Table 4 below shows the path between deductions from the graph in practical Physics and SSCE Physics achievement in the direction shown by the arrow. The estimated beta-weight of the path is 0.17.

Table 4: The coefficient of linear regression between the path: Physics achievement and deductions from graph in practical Physics in the combined model.

Linear Regression path	Beta-weight
SSCE Physics Achievement and Deductions from graph in practical Physics	0.17

The value of the path coefficient shows that a low positive relationship between SSCE Physics Achievement and deductions from the graph in practical Physics exists.

Hypotheses:

$H0_1$: There is no significant difference between path coefficients in the observed and expected structural equation model of male Physics students (p .05).

Table 5 shows the result of the Chi-square test for male Physics students' model. The Chi-square has a value of 435.172. The degree of freedom in the model, associated to the Chi-square is 425. The significance attached to the stated Chi-square and degree of freedom values for the model has a value of .356.

Table 5: The Chi-square value for male Physics students' model.

Chi-square	df	Significance	Decision
435.172	425	.356	Accept null hypothesis

Since the significance value of .356 is more than .05 level of significance set for the study, the null hypothesis which posits that there is no significance difference between observed and expected path coefficients in the structural equation model is accepted. This means that both sets of data (theoretical and empirical) fit the path model based on the hypothesis of similar path coefficients. The Chi-square significance shows that the regression weights in the male students' model were valid enough to be accepted.

$H0_2$: There is no significant difference of path coefficients in the observed and expected structural equation model of female Physics students (p .05).

Table 6 shows the result of the Chi-square test for female Physics students' model. The Chi-square has a value of 477.982. The degree of freedom in the model, associated to the Chi-square is 425. The significance attached to the stated Chi-square and degree of freedom values for the model has a value of .068.

Table 6: The Chi-square value for female Physics students' model.

Chi-square	df	Significance	Decision
477.982	425	.068	Accept null hypothesis

Since the significance value of .068 is more than .05 level of significance set for the study, the null hypothesis which posits that there is no significance difference between observed and expected path coefficients in the structural equation model is accepted. This means that both sets of data (theoretical and empirical) fit the path model based on the hypothesis of similar path coefficients. The Chi-square significance shows that the regression weights in the female students' model were valid enough to be accepted.

$H0_3$: There is no significant difference between male and female students' structural equation models' path coefficients (p .05).

Table 7 shows the result of the Chi-square test for the combined male and female Physics students' model. The Chi-square has a value of 409.621. The degree of freedom in the model, associated to the Chi-square is 425. The significance attached to the stated Chi-square and degree of freedom values for the model has a value of .695.

Table 7: The Chi-square value for the combined Physics students' model.

Chi-square	df	Significance	Decision
409.621	425	.695	Accept null hypothesis

Since the significance value of .695 is more than .05 level of significance set for the study, the null hypothesis which posits that there is no significance difference between observed and expected path coefficients in the structural equation model is accepted. This means that both sets of data (theoretical and empirical) fit the path model based on the hypothesis of similar path coefficients. The Chi-square significance shows that the regression weights in the combined male and female students' model were valid enough to be accepted.

Discussion of the Findings

The findings of the study showed that a positive linear relationship exist between SSCE Physics achievement and generation of readings from equations in practical Physics. Basic practical Physics mathematical operations, graphing skills and deductions from the graph have positive linear relationship with SSCE Physics achievement. Basic practical Physics mathematical operation has the highest linear relationship with SSCE Physics achievement in terms of standard deviation units in the model. Graphing skill is followed by generation of readings from equations in terms of explaining the variance in SSCE Physics achievement. The least positive contributor in the explanation of the variance in SSCE Physics achievement in the model was deduction from the graph. The findings are in tandem with earlier studies (Thompson & Steinberg, 2014; Smith, 2015; Ramirez &Ramos, 2016; Kapucu, et al., 2016; Johnson et al., 2017; Wu et al., 2017; Chen & Klahr, 2018; Brown & Lee, 2019; Hu et al., 2019;; Li et al., 2021; Chen, et al., 2021) which stated that a positive linear relationship existed between mathematical knowledge and Physics achievement. However, the present finding is at variance with the finding of Winter and Airey (2022), who indicated that mathematical knowledge was not a factor in the prediction of Physics achievement. None of the studies reviewed utilized structural equation model (SEM) to test the linear relationships between the mathematical concepts applied to practical Physics and SSCE Physics achievement. SEM is a hybrid model for studying the relationships between latent and observed variables as a whole. The model provided a good fit to the data. It was also found out that the path coefficients were similar for male and female sub-samples. This means that the result of this study can be applied to both male and female sub-samples.

Conclusion and Recommendations

Mathematical knowledge is positively related to Physics achievement. It is recommended that Physics teachers should emphasize mathematical knowledge in practical Physics classes for attainment of sustainable development goals.

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